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Lorentz and CPT violation: a simple neutrino-oscillation model

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We present a three-parameter model for three flavors of massless left-handed neutrinos with Fermi-point splitting and tri-maximal mixing angles. One of these parameters is the T–violating phase δ, for which the experimental results from K2K and KamLAND appear to favor a nonzero value. Future experiments, in particular MINOS, will be able to test this simple model. Possible implications for neutrino factories are briefly discussed.

1. INTRODUCTION

The experimental results on neutrino oscillations (as discussed by other talks at this Conference) suggest the following pattern of mixing angles:

$$\underline{\theta}_{21} \approx \underline{\theta}_{32} \approx \pi/4 \,, \quad \underline{\theta}_{13} \approx 0 \,, \tag{1}$$

assuming the validity of the mass-difference mechanism [1]. But, perhaps, one angle need not be small if there are other mechanisms operative?

In fact, there is the possibility that Lorentz invariance is not a fundamental symmetry but an emergent phenomenon [2]. Massless (or nearly massless) neutrinos could then provide us with a window on "really new physics."

In this talk, we discuss an idea from condensed-matter physics, namely Fermi point splitting by a quantum phase transition [3,4]. (Fermi points $\mathbf{p}_n$ are points in three-dimensional momentum space at which the energy spectrum of the fermionic quasi-particle has a zero.) The neutrino-oscillation model considered [5,6] is the simplest one possible with *all* mixing angles large and mass differences vanishing *exactly*. Needless to say, this model may be only a first approximation.

Note that the idea of neutrino oscillations from Fermi-point splittings is orthogonal to the suggestion of having CPT–violating masses to explain LSND (see, e.g., Ref. [7] and references therein).

2. FERMI-POINT-SPLITTING ANSATZ

In the limit of vanishing Yukawa couplings, the chiral fermions of the Standard Model may still have Fermi-point splittings in their dispersion law,

$$\left(E_{a,f}(\mathbf{p})\right)^2 = \left(c\,|\mathbf{p}| + b_{0a}^{(f)}\right)^2 . \tag{2}$$

Here, a labels the 16 types of massless left-handed Weyl fermions in the Standard Model (with a hypothetical left-handed antineutrino included) and f distinguishes the 3 known fermion families.

An example of Fermi-point splitting is given by the following factorized *Ansatz* [4]:

$$b_{0a}^{(f)} = Y_a\, b_0^{(f)} \,, \tag{3}$$

with Y_a the Standard Model hypercharges of the left-handed fermions. For this special pattern, the induced electromagnetic Chern–Simons term cancels out exactly. This allows for b_0 values very much larger than the experimental upper limit on the Chern–Simons energy scale, which is of the order of 10^{-33} eV [8].

Independent of the particular pattern of Fermi-point splitting, the dispersion law of a massless left-handed neutrino can be written as

$$\left(E_{\nu_L,f}(\mathbf{p})\right)^2 = \left(c\,|\mathbf{p}| - b_0^{(f)}\right)^2 . \tag{4}$$

The right-handed antineutrino is assumed to have the same dispersion law as (4) but with a plus sign in front of $b_0^{(f)}$ (the case with a minus sign is also discussed in Ref. [6]).

More generally, one may consider for large momenta $|\mathbf{p}|$:

$$E(\mathbf{p}) \sim c\,|\mathbf{p}| \pm b_0 + \frac{m^2\,c^4}{2|\mathbf{p}|c} + \mathrm{O}\left(|\mathbf{p}|^{-2}\right) . \tag{5}$$

The energy change from a nonzero b_0 always dominates the effect from mc^2 for large enough $|\mathbf{p}|$. In order to search for Fermi-point splitting, it is therefore preferable to use neutrino beams with the highest possible energy.

In this talk, we set all neutrino masses to zero. Let us emphasize that this is only a simplifying assumption and that there may very well be additional mass terms, as in Eq. (5). However, with both mass terms and Fermi-point splittings present, there is a multitude of mixing angles and phases to consider, which is the reason to leave the masses out in an exploratory analysis.

3. THREE-PARAMETER MODEL

The flavor states $|A\rangle, |B\rangle, |C\rangle$ and the left-handed propagation states $|1\rangle, |2\rangle, |3\rangle$ with dispersion law (4) are related by a unitary 3×3 matrix U:

$$
\begin{pmatrix} |A\rangle \\ |B\rangle \\ |C\rangle \end{pmatrix} = U \begin{pmatrix} |1\rangle \\ |2\rangle \\ |3\rangle \end{pmatrix}. \tag{6}
$$

The standard parametrization of U has one phase $\delta \in [0, 2\pi)$, and three mixing angles $\theta_{21}, \theta_{32}, \theta_{13} \in [0, \pi/2]$.

In order to emphasize the difference with the current paradigm (1) for the mixing angles $\underline{\theta}_{ij}$ associated with mass terms, we take the mixing angles from (6) to be *tri-maximal*:

$$
\theta_{21} = \theta_{32} = \arctan 1 = \pi/4, \tag{7a}
$$
$$
\theta_{13} = \arctan \sqrt{1/2} \approx \pi/5. \tag{7b}
$$

These particular values maximize, for given phase δ, the T–violation (CP–nonconservation) measure $J \equiv \frac{1}{8} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{21} \sin 2\theta_{32} \sin\delta$ of Ref. [9].

The Fermi-point-splitting energies $b_0^{(f)}$ are assumed to be positive and to increase with f, giving rise to two positive parameters:

$$
B_0 = b_0^{(2)} - b_0^{(1)}, \tag{8a}
$$
$$
r = \left(b_0^{(3)} - b_0^{(2)} \right) / \left(b_0^{(2)} - b_0^{(1)} \right). \tag{8b}
$$

All in all, the model [6] has three parameters:

- the basic energy-difference scale B_0,

- the ratio r of the two energy steps Δb_0,

- the T–violating phase δ.

This model will be called the "simple" Fermi-point-splitting model in the following (a more general Fermi-point-splitting model would have arbitrary mixing angles θ_{ij}).

4. OSCILLATION PROBABILITIES

For large enough neutrino energy (that is, $E_\nu \geq \max [\, b_0^{(f)}]$), the tri-maximal model gives neutrino oscillation probabilities

$$
P(X \to Y), \quad \text{for } X, Y \in \{A, B, C\}, \tag{9}
$$

in terms of the dimensionless distance

$$
l \equiv B_0 \, L/(h\,c) \tag{10}
$$

and the other two model parameters, r and δ. With the assumed dispersion laws, the same probabilities hold for antineutrinos,

$$
P(\bar{X} \to \bar{Y}) = P(X \to Y). \tag{11}
$$

For the model probabilities, the time-reversal asymmetry between A–type and C–type neutrinos is given by

$$
a_{CA}^{(\mathrm{T})} \equiv \frac{P(A \to C) - P(C \to A)}{P(A \to C) + P(C \to A)} \propto \sin\delta, \tag{12}
$$

whereas the CP–asymmetry vanishes identically.

5. PARAMETERS AND PREDICTIONS

5.1. General predictions

Two general predictions [5] of the Fermi-point-splitting mechanism of neutrino oscillations are:

- *undistorted* energy spectra for the reconstructed ν_μ energies in, for example, the current K2K experiment and the future MINOS experiment;

- survival probabilities close to 1 for *all* reactor experiments at $L \approx 1\,\mathrm{km}$ (e.g., CHOOZ and double-CHOOZ), at least up to an accuracy of order $(\Delta b_0 \, L/\hbar c)^2 \approx 10^{-4}$ for $\Delta b_0 \approx 2 \times 10^{-12}$ eV (see below).

Table 1
Lengths [km] and time-reversal asymmetries $a_{\mu e}^{(\mathrm{T})}$ for selected model parameters $B_0\,[10^{-12}\,\mathrm{eV}]$ and r, with phase $\delta = \pi/4$ (mod π) and identifications (14ab).

B_0	r	λ	L_{magic}	$a_{\mu e}^{(\mathrm{T})}\,(L_{\mathrm{magic}})$	L'_{magic}	$a_{\mu e}^{(\mathrm{T})}\,(L'_{\mathrm{magic}})$
0.72	1	1724	386	$+89\,\%$	1338	$-89\,\%$
1.04	1/2	2381	357	$+99\,\%$	2024	$-99\,\%$
1.29	1/4	3846	395	$+81\,\%$	3451	$-81\,\%$

Both predictions hold only if mass-difference effects from the generalized dispersion law (5) are negligible compared to Fermi-point-splitting effects. With (anti)neutrino energies in the MeV or GeV range, this corresponds to $\Delta m^2 \lesssim 10^{-6}\,\mathrm{eV}^2/c^4$ or $\Delta m^2 \lesssim 10^{-3}\,\mathrm{eV}^2/c^4$, respectively.

5.2. Preliminary parameter values

Comparison of the model probabilities and the *combined* K2K and KamLAND results gives the following "central values" [6]:

$$B_0 \;\approx\; 10^{-12}\,\mathrm{eV}\,, \tag{13a}$$

$$r \;\approx\; 1/2\,, \tag{13b}$$

$$\delta \;\approx\; \pi/4 \;\;\text{or}\;\; 5\pi/4\,, \tag{13c}$$

with identifications

$$\bigl(|A\rangle\,,|B\rangle\,,|C\rangle\bigr) = \bigl(|\nu_e\rangle\,,|\nu_\tau\rangle\,,|\nu_\mu\rangle\bigr)\Big|^{\delta\approx\pi/4}, \tag{14a}$$

$$\bigl(|A\rangle\,,|B\rangle\,,|C\rangle\bigr) = \bigl(|\nu_e\rangle\,,|\nu_\mu\rangle\,,|\nu_\tau\rangle\bigr)\Big|^{\delta\approx 5\pi/4}. \tag{14b}$$

Detection of an interaction event (e.g., μ^- decay) is needed to decide between the options (14ab).

5.3. Specific predictions

Detailed model predictions can be found in Ref. [6], in particular figures relevant to MINOS and T2K (JPARC–SK). If the model has any validity, MINOS should be able to reduce the range of r values compared to the range allowed by current K2K data.

Table 1 gives the wavelength λ, the distance L_{magic} which maximizes the time-reversal asymmetry $a_{\mu e}^{(\mathrm{T})}$ from Eq. (12), and the other magic distance L'_{magic} which minimizes this T–asymmetry. Observe that $L_{\mathrm{T2K}} = 295$ km is of the same order of magnitude as L_{magic}, which would make having both ν_e and ν_μ beams from JPARC especially interesting.

6. OUTLOOK

We propose to use the following checklist:

- *equal survival probabilities $P(\nu_\mu \to \nu_\mu)$ for the low- and high-energy beams of MINOS?*

- *appearance probability $P(\nu_\mu \to \nu_e)$ from MINOS above a few percent?*

- *consistent fit of the (simple) Fermi-point-splitting model to the combined data from K2K, MINOS, and ICARUS/OPERA?*

If this more or less works out, one would have to reconsider the future options based on the relevant *energy-independent* length scales of the (simple) Fermi-point-splitting model [cf. Table 1] or those of an extended version with additional mass terms ($\Delta m^2 \approx 10^{-4}\,\mathrm{eV}^2/c^4$?). These future options include beta beams and neutrino factories. But, first, let's see what MINOS finds ...

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